**Journal of Electrical Engineering, Electronics, Control and Computer Science – JEEECCS, Volume 10, Issue 35, pages 21-30, 2024**

# Discussion and correction of the equivalent circuit of induction motor and its torque generation

Ching-Yu Lin[1]
China steel corporation
Electrical & Control Dept.
Kaohsiung, Taiwan
cylintw24@gmail.com

Po-Yuan Chen[2]
CECI Engineering Consultants,
KC Department
Taipei, Taiwan
pychen@ceci.com.tw

Su-Che Wu[3]
CECI Engineering Consultants
Rapid Transit Eng. Department
Kaohsiung, Taiwan
alexstwu0420@gmail.com

**Abstract - To explore the actual electrical characteristics of the stator and rotor inside an induction motor, the magnetomotive force of each phase winding on both sides is expressed using a reasonable mathematical formula. According to Faraday's induction law, the counter electromotive force and induced voltage equations for both sides are derived. By substituting these mathematical equations into an Excel calculation worksheet, it is found that the electrical frequency cycles on both sides differ and their changing rules are identified. The voltages and currents of different frequency cycles on both sides are compared using the same unit average frequency periodic root mean square value. The comparison ratio provides the rotor equivalent impedance reference to the stator side, leading to the deduction of the correct induction motor equivalent circuit. The new equivalent circuit, verified through a series of circuit calculation analysis results, complies with energy conservation and Thevenin's theorem. From the overall energy distribution, it is observed that the active power on both sides' accounts only for the copper loss and waste heat that cannot perform work. Conversely, the reactive power generates the magnetic field torque necessary to drive the load. Additionally, special periodic wave changes in the actual torque are discovered.**

***Keywords-magnetomotive force; Faraday's induction law; unit average frequency periodic root mean square value (uarms); equivalent circuit; Thevenin's theorem***

## Ⅰ. INTRODUCTION

To simplify the process in this study, a three-phase balanced motor circuit is considered, and some chapters are discussed only in single phase. The rotor winding is assumed to have the same number of turns per phase as the stator winding [1]. The equivalent impedance of the rotor is referred to the stator end and converted to the true impedance of the rotor according to the real winding turn ratio if required.

Determining the wave frequency and period of the phase winding of the stator counter electromotive force (EMF) and rotor-induced voltage and current at a certain slip speed is challenging. The stator counter EMF, generated by the variations in flux linkage of the stator phase winding, is denoted as Esa [V]. To address this, an Excel calculation worksheet was designed for deduction and analysis.

When it is found that the induced voltage and current of the rotor side have different frequencies and periods on the stator side, the same unit average frequency period root mean square value (uarms) calculation method is used. This method obtains the individual voltage and current root mean square values at the stator side (rms) and rotor side (uarms). The equivalent circuit and each impedance value of the induction motor are derived, and the overall distribution of active power and reactive power is obtained by calculation.

Finally, this study uses data from a factory test report of an induction motor from many years ago to analyze and separate the impedance values of the stator and rotor and the distribution of the overall energy input from the power source. Additionally, the torque generation of the induction motor is based on the theory of the magnetic field. The repulsive and attractive forces that change periodically with time are also designed using an Excel computer calculation worksheet for deduction and analysis.

## Ⅱ. MATHEMATICAL FORMULA OF MAGNETOMOTIVE FORCE FOR STATOR AND ROTOR

The magnetic axis of each stator phase winding is fixed. The magnetomotive force (MMF) generated by the stator phase windings varies with the sine wave of the stator's external power supply current. Each phase has a 120° angle difference. The three phases are named Fsa(θ, t), Fsb(θ-120°, t), and Fsc(θ-240°, t). However, the rotor experiences speed slippage due to different loads. This slippage results in different frequencies and periods of the magnetic flux passing through the air gap to the rotor poles. The relative movement between the stator and rotor causes variations in the corresponding magnetic pole fluxes.

Despite these differences, the stator and rotor form a closed magnetic path. Although the frequencies and periods differ, the average flux per unit time remains constant. Therefore, the flux for stator phase-a winding (Φsa) equals the flux for rotor phase-a winding (Φra). Both Φsa and Φra are measured in Weber (Wb).

*A. The MMF Generated By Each Phase Winding Of The Stator*

The motor is an inductive load. Thus, the phase current lags behind the phase voltage by an angle θ. Assume that the starting point is on the axis of phase a. At any time t, the MMF generated by the a, b, and c phase magnetic poles (see Fig. 1) is as follows:

Each phase is separated by 120°. The maximum MMF for the three phases are Nsa · Isa-max, Nsb · Isb-max, and Nsc · Isc-max, respectively. Theoretically, the path reluctance (Rkx), the number of turns (Nsx), and the maximum phase current (Isx-max) are equal for all three phases. Therefore, all three phases can be represented by Fmax.

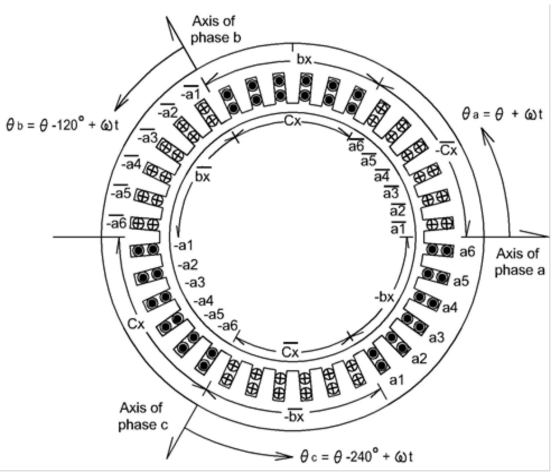


Figure 1. 2-pole 3-phase stator winding diagram

$$F_{sa}(\theta,t) = R_{ka}\cdot\Phi_{sa} = F_{max}\cdot Cos(\theta+\omega t) = N_{sa}\cdot I_{sa-max}\cdot Cos(\theta+\omega t) \quad (1)$$

$$F_{sb}(\theta-120°,t) = R_{kb}\cdot\Phi_{sb} = F_{max}\cdot Cos(\theta-120°+\omega t) = N_{sb}\cdot I_{sb-max}\cdot Cos(\theta-120°+\omega t) \quad (2)$$

$$F_{sc}(\theta-240°,t) = R_{kc}\cdot\Phi_{sc} = F_{max}\cdot Cos(\theta-240°+\omega t) = N_{sc}\cdot I_{sc-max}\cdot Cos(\theta-240°+t) \quad (3)$$

*B. The MMF at each phase winding of the rotor with a speed slippage(S)*

$$F_{ra}(\theta,t) = R_{ka}\cdot\Phi_{ra} = F_{max}\cdot Cos(\theta+\omega t)\cdot Cos(1-S)\omega t = N_{sa}\cdot I_{sa-max}\cdot Cos(\theta+\omega t)\cdot Cos(1-S)\omega t \quad (4)$$

$$F_{rb}(\theta-120°,t) = R_{kb}\cdot\Phi_{rb} = F_{max}\cdot Cos(\theta-120°+\omega t)\cdot Cos(1-S)\omega t = N_{sb}\cdot I_{sb-max}\cdot Cos(\theta-120°+\omega t)\cdot Cos(1-S)\omega t \quad (5)$$

$$F_{rc}(\theta-240°,t) = R_{kc}\cdot\Phi_{rc} = F_{max}\cdot Cos(\theta-240°+\omega t)\cdot Cos(1-S)\omega t = N_{sc}\cdot I_{sc-max}\cdot Cos(\theta-240°+\omega t)\cdot Cos(1-S)\omega t \quad (6)$$

*C. The resultant MMF at the axis of each phase winding of the stator: (converted by trigonometric function)*

$$F_{sa-resultant}(\theta,t) = F_{max}\cdot Cos(\theta+\omega t)\cdot Cos(0°) + F_{max}\cdot Cos(\theta-120°+\omega t)\cdot Cos(-120°) + F_{max}\cdot Cos(\theta-240°+\omega t)\cdot Cos(-240°) = F_{max}\cdot[(1/2)\cdot Cos(\theta+\omega t)+(1/2)\cdot Cos(\theta+\omega t)] + F_{max}\cdot[(1/2)\cdot Cos(\theta+\omega t)+(1/2)\cdot Cos(\theta-240°+\omega t)] + F_{max}\cdot[(1/2)\cdot Cos(\theta+\omega t)+(1/2)\cdot Cos(\theta-480°+\omega t)] = (3/2)F_{max}\cdot Cos(\theta+\omega t) \quad (7)$$

$$F_{sb-resultant}(\theta-120°,t) = F_{max}\cdot Cos(\theta-120°+\omega t)\cdot Cos(0°) + F_{max}\cdot Cos(-240°+\omega t)\cdot Cos(-120°) + F_{max}\cdot Cos(\theta+\omega t)\cdot Cos(-240°) = (3/2)F_{max}\cdot Cos(\theta-120°+\omega t) \quad (8)$$

$$F_{sc-resultant}(\theta-240°,t) = F_{max}\cdot Cos(\theta-240°+\omega t)\cdot Cos(0°) + F_{max}\cdot Cos(\theta+\omega t)\cdot Cos(-120°) + F_{max}\cdot Cos(\theta-120°+\omega t)\cdot Cos(-240°) = (3/2)F_{max}\cdot Cos(\theta-240°+\omega t) \quad (9)$$

*D. The resultant MMF at the axis of each phase winding of the rotor*

$$F_{ra-\mathrm{resultant}}(\theta,t) = \left(\tfrac{3}{2}\right)F_{max}\cdot Cos(\theta+\omega t)\cdot Cos(1-S)\omega t \quad (10)$$

$$F_{rb-\mathrm{resultant}}(\theta-120°,t) = \left(\tfrac{3}{2}\right)F_{max}\cdot Cos(\theta-120°+\omega t)\cdot Cos(1-S)\omega t \quad (11)$$

$$F_{rc-\mathrm{resultant}}(\theta-240°,t) = \left(\tfrac{3}{2}\right)F_{max}\cdot Cos(\theta-240°+\omega t)\cdot Cos(1-S)\omega t \quad (12)$$

## III. VOLTAGE AND CURRENT CHARACTERISTICS OF STATOR AND ROTOR

When the load current of an induction motor passes through the stator phase winding, it generates magnetomotive force (MMF) and causes a voltage drop across the phase winding. Additionally, it produces a counter electromotive force (EMF). This EMF, denoted as Esa for the stator and Era for the rotor (with the other two phases named Erb and Erc), is the time derivative of the phase magnetic pole flux linkage (e = dλ/dt). Here, λsa = Nsa · Φsa and λra = Nra · Φra [2&6]. The voltage drop across the phase winding and the counter EMF are connected in series and equal to the external power supply terminal voltage "(14)", as shown in Fig. 2 [1].

At the same time, the time-varying MMF from the stationary stator winding reaches the corresponding rotor magnetic pole through the air gap. Due to the relative movement between the stator and rotor, the MMF of the rotor phase windings varies more complexly ["(4)"~"(6)"]. This causes the MMF-induced voltage, current frequency, and period of the rotor phase winding to differ from those of the stator phase winding at the power supply end. Additionally, the varied MMF-induced voltage and current, which have different frequencies and periods under specific rotational slip, result in changes to the impedance value of the rotor phase winding.

In fact, the current Isa at the stator terminal input decomposes into two components: the load component I1sa (stator coil current) and the magnetizing component IΦa, shown as Fig. 2. This can be expressed as:

$$\text{Isa} = \text{I1sa} + \text{I}\Phi \quad (13)$$

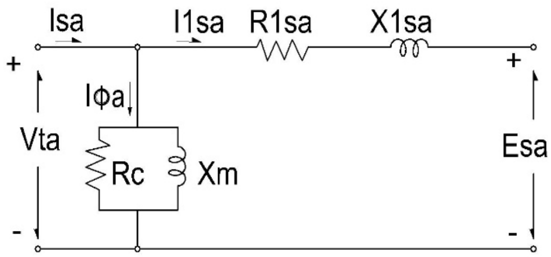


Figure 2. Three-phase induction motor a-phase stator equivalent circuit diagram

$$V_{ta} = I1_{sa}(R1_{sa}+X1_{sa}\, i) + E_{sa} \quad (14)$$

$V_{ta}$ is the stator AC power phase-a voltage, ($i^2 = -1$)
$I_{sa}$ is the stator AC power supply phase-a current
R1sa is the stator winding resistance
$X1_{sa}$ is the stator winding reactance
Rc is the magnetizing resistance
$X_m$ is the magnetizing inductance
Esa is the EMF of the stator phase-a.

$$E_{sa} = d\lambda_{sa}/dt = N_{sa} \cdot d\Phi_{sa}/dt$$
$$= N_{sa} \cdot d[N_{sa} \cdot I_{sa-max} \cdot Cos(\theta + \omega t)]/dt$$
$$= -\omega \cdot N_{sa}^2 \cdot I_{sa-max} \cdot Sin(\theta + \omega t) \quad (15)$$

When the EMF (Esa) is generated, it corresponding rotor phase-a winding also generates an induced-voltage (Era). Here,

$$E_{ra} = d\lambda_{ra}/dt = N_{ra} \cdot d\Phi_{ra}/dt$$
$$= N_{ra}\, d\begin{bmatrix} N_{ra} \cdot I_{sa-max} \cdot Cos(\theta + \omega t) \cdot \\ Cos(1-S)\omega t \end{bmatrix}/dt$$
$$= [-\omega \cdot N_{ra}^2 \cdot I_{sa-max}] \cdot [Sin(\theta + \omega t) \cdot Cos(1-S)\omega t + (1-S) \cdot Cos(\theta + \omega t) \cdot Sin(1-S)\omega t] \quad (16)$$

## A. *Frequency And Period Of Stator And Rotor Voltage And Current*

The stator side is directly supplied by an external public power supply. If its voltage and current have a frequency of 60Hz and a period width of 2π, then the stator phase winding voltage drop, counter EMF and current will have the same frequency and period. According to "(1)" ~ "(3)" and "(15)", both voltage and current are sine wave functions. Therefore, their frequency and period are the same as the external public power supply. On the rotor side, the MMF generated by the stator phase winding reaches the corresponding magnetic pole of the rotating rotor through the air gap. Due to the change in the amount of magnetic flux between the two relative motions, the magnetic flux induction voltage, current frequency and period of the rotor phase winding as described by "(4)" ~ "(6)" and "(16)" are significantly different. Based on the continuous situation analysis of various rotor rotation slip speeds and the actual rotor rotation timing angular position changes, the results are summarized in Table 1. and Table 2.

Table 1. Voltage and current of stator and rotor at different slip (S) speeds, Frequency and period variation

| Slip (S) | Stator (Isa &Esa ) | Rotor (Ira& Era) | Stator / Rotor | |
|---|---|---|---|---|
| | $f_s/C_{w-s}$ | $f_r/C_{w-r}$ | $(f_s/f_r)$ | $(C_{w-s}/C_{w-r})$ |
| 1 | 60Hz / 2π | 60Hz / 2π | 1 | 1 |
| 1/6 | 60Hz / 2π | (2-1/6)60Hz /12π | 1/(2-S) | S |
| 1/6.3 | 60Hz / 2π | (2-1/6.3)60Hz / 126π | 1/(2-S) | S |
| 1/63 | 60Hz / 2π | (2-1/63)60Hz /126π | 1/(2-S) | S |
| 1/120 | 60Hz / 2π | (2-1/120)60Hz /240π | 1/(2-S) | S |
| 0 | 0 | 0 | 0 | 0 |

Remark: fs/Cw-s --Frequency/Cycle width of current(I1sa) and voltage (Esa) at Stator windings
fr/Cw-r --Frequency/Cycle width of current (Ira) and voltage (Era) at rotor windings

Table 2. The square integral of waveform functions of MMF , EMF and induced voltage of stator and rotor

| Slip(S) | Fsa-rms | Fra-uarms | Esa-rms | Era-uarms |
|---|---|---|---|---|
| | *A* | *B* | *C* | *D* |
| 1 | 180 | 180 | 180 | 180 |
| 1/6 | 1080 | 540 | 1080 | 915 |
| 1/6.3 | 11340 | 5670 | 11340 | 9682.86 |
| 1/63 | 11340 | 5670 | 11340 | 11161.43 |
| 1/120 | 21600 | 10800 | 21600 | 21420.75 |
| 0 | 0 | 0 | 0 | 0 |

Remark:

$$A = \sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} [Cos(\theta + \omega t)]^2$$

$$B = \sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} [Cos(\theta + \omega t) \cdot Cos(1-S)\omega t]^2$$

$$C = \sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} [Sin(\theta + \omega t)]^2$$

$$D = \sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} [Sin(\theta + \omega t) \cdot Cos(1-S)\omega t + (1-S) \cdot Cos(\theta + \omega t) \cdot Sin(1-S)\omega t]^2$$

## B. *The Mutual Ratio Of Voltage And Current Between The Stator And Rotor*

Due to the change in MMF between the rotor phase winding and the corresponding phase winding of the stator, the rotor induced voltage and current have different in frequencies and periods compared to those on the stator winding side. To obtain the actual comparison value of the voltage and current on both sides, the root mean square (rms) voltage value and current value over the same average unit frequency period must be compared. On the stator side, it is the general root mean square (rms) of Sine wave with a frequency of 60Hz and a period width of 2π. On the rotor side, the same average unit frequency period root mean square (uarms) as the stator is used.

1) *The ratio of the stator phase winding counter EMF to the rotor induced voltage:*

According to "(15)",

$$E_{sa-rms} = [-\omega \cdot N_{sa}^2 \cdot I_{sa-max}] \cdot \sqrt{\left\{\sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} \frac{[Sin(\theta+\omega t)]^2}{(2/S)}/180\right\}} = [-\omega \cdot N_{sa}^2 \cdot I_{sa-max}] \cdot \sqrt{\{C/(2/S)/180\}} \quad (17)$$

Bring the results obtained from the calculation worksheet of Table 2 , e.g. When $S = 1/6$, the C value is 1080.

$$E_{sa-rms} = [-\omega \cdot N_{sa}^2 \cdot I_{sa-max}] \cdot \sqrt{(1/2)} \quad (18)$$

From "(16)",

$$E_{ra-uarms} = [-\omega \cdot N_{ra}^2 \cdot I_{sa-max}] \cdot \sqrt{\{\sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} [\boldsymbol{Sin(\theta+\omega t)\cdot Cos(1-S)\omega t + (1-S)\cdot Cos(\theta+\omega t)\cdot Sin(1-S)\omega t}]^2/(2/S)/[(1/S).180]\}} = [-\omega \cdot N_{sa}^2 \cdot I_{sa-max}] \cdot \sqrt{\{D/(2/S)/[(1/S)/180]\}} \quad (19)$$

Bring the results obtained from the calculation worksheet of Table 2., e.g. When $S = 1/6$, the D value is 915.

$$E_{ra-uarms} = [-\omega \cdot N_{ra}^2 \cdot I_{sa-max}] \cdot \sqrt{\left(\frac{S}{2}\right)} \cdot \sqrt{\left(\frac{1}{2}\right)} \cdot \sqrt{\left(\frac{\frac{915}{3}}{180}\right)} = [-\omega \cdot N_{ra}^2 \cdot I_{sa-max}] \cdot \sqrt{\left(\frac{S}{2}\right)} \cdot \sqrt{\left(\frac{1}{2}\right)} \cdot \sqrt{[(1-S)^2+1]} \quad (20)$$

So,

$$\frac{E_{sa-rms}}{E_{ra-uarms}} = \sqrt{\left(\frac{2}{S}\right)} \cdot \sqrt{\left\{\frac{1}{[(1-S)^2+1]}\right\}} \quad (21)$$

2) *The relative ratio of the stator phase winding current to the rotor phase winding induced current:*

According to "(1)",

$$F_{sa-rms} = [N_{sa} \cdot I_{sa-max}] \cdot \sqrt{\left\{\sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} [Cos(\theta+\omega t)]^2/(2/S)/180\right\}} \cdot = [N_{sa} \cdot I_{sa-max}] \cdot \sqrt{\{A/(2/S)/180\}} \quad (22)$$

Bring the results obtained from the calculation worksheet of Table 2, e.g. When $S = 1/6$, the *A* value is 1080.

$$F_{sa-rms} = [N_{sa} \cdot I_{sa-max}] \cdot \sqrt{\left(\frac{1}{2}\right)} = N_{sa} \cdot I_{sa-rms} \quad (23)$$

From "(2)",

$$F_{ra-uarms} = [N_{ra} \cdot I_{sa-max}] \cdot \sqrt{\left\{\sum_{\omega t=0}^{\omega t=\frac{2\pi}{S}} \frac{[Cos(\theta+\omega t)\cdot Cos(1-S)\omega t]^2}{(2/S)}/\left[\left(\frac{1}{S}\right)\cdot 180\right]\right\}} = [N_{ra} \cdot I_{sa-max}] \cdot \sqrt{\{B/(2/S)/[(1/S)\cdot 180]\}} \quad (24)$$

Bring the results obtained from the calculation worksheet of Table 2, e.g. When $(S = 1/6)$ , the *B* value is 540.

$$F_{ra-uarms} = [N_{ra} \cdot I_{sa-max}] \cdot \sqrt{\left(\frac{S}{2}\right)} \cdot \sqrt{\left(\frac{1}{2}\right)} = \left[N_{ra} \cdot I_{sa-max} \cdot \sqrt{\left(\frac{2}{S}\right)} \cdot \sqrt{\left(\frac{S}{2}\right)}\right] \cdot \sqrt{\left(\frac{S}{2}\right)} \cdot \sqrt{\left(\frac{1}{2}\right)} = \left[N_{ra} \cdot I_{ra-max} \cdot \sqrt{\left(\frac{S}{2}\right)}\right] \cdot \sqrt{\left(\frac{S}{2}\right)} \cdot \sqrt{\left(\frac{1}{2}\right)} = \left[N_{ra} \cdot I_{ra-uarms} \cdot \sqrt{\left(\frac{S}{2}\right)}\right] \quad (25)$$

Because , $F_{sa-rms} = F_{ra-uarms}$,
So,

$$I_{sa-rms} = I_{ra-uarms} \cdot \sqrt{\left(\frac{S}{2}\right)}$$

$$I_{sa-rms}/I_{ra-uarms} = \sqrt{\left(\frac{S}{2}\right)} \quad (26)$$

## IV. EQUIVALENT CIRCUIT DERIVATION AND ENERGY CONSUMPTION ANALYSIS

### *A. Equivalent Circuit Derivation*

When the induction motor operates stably under a normal load, there is a fixed slip(S) speed. When S=1, the rotor of the induction motor is blocked and locked. At this moment, the frequency and period of the voltage and current on both the stator and rotor sides are exactly the same as the external input power supply as showing in Table 1& Table 2.

Therefore, the relative ratio of the counter EMF to induced voltage of the windings and the current between them is also 1. The number of turns of the windings on both sides is equal, i.e., $E_{sa-rms} = E_{ra-rms}$ , $I_{1sa-rms} = I_{ra-rms}$ . Consequently, $Z_{sa} = Z_{ra}$.

In this situation, the stator and rotor windings of the induction motor are like a static transformer with two windings. When the number of winding turns on both sides is equal and the secondary winding of the transformer is short-circuited, the equivalent circuit is directly referenced from the impedance of the rotor side to the stator side shown as Fig. 3a.

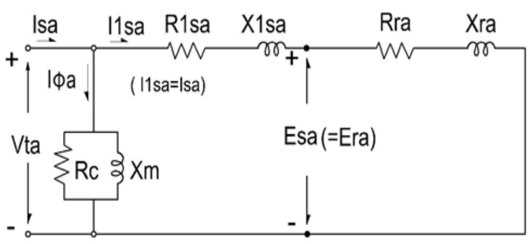


Figure 3a. Three-phase induction motor a-phase stator equivalent circuit diagram (When $S$=1)

However, when 0< $S$ <1; According to"(21)" & "(26)", we get,

$$\frac{E_{sa-rms}}{I_{sa-rms}} = \left\{ \left\{ \frac{\sqrt{\left(\frac{2}{S}\right)} \cdot \sqrt{\left[\frac{1}{[(1-S)^2+1]}\right]}}{\sqrt{\left(\frac{S}{2}\right)}} \right\} \cdot [E_{ra-uarms} / I_{ra-uarms}] \right\} \tag{27}$$

So,

$$Z_{sa} = \left\{ \left(\frac{2}{S}\right) \cdot \sqrt{\left\{\frac{1}{[(1-S)^2+1]}\right\}} \cdot Z_{ra} \right\}$$
$$= \left\{ \left(\frac{2}{S}\right) \cdot \sqrt{\left\{\frac{1}{[(1-S)^2+1]}\right\}} \cdot [R_{ra} + (2-S)X_{ra}i] \right\}$$
$$= K_{cw} \cdot K_s \cdot \left[R_{ra} + K_f \cdot X_{ra}i\right] \tag{28}$$

The above "(28)" represents the change of the rotor side phase winding impedance ( $Z_{ra}$ ) of an induction motor in stable operation and is compared with the equivalent impedance ($Z_{sa}$) referenced to the stator side. Among them:

- $K_{cw} = (2/S)$ is the period factor of the rotor winding impedance referenced to the stator.
- $K_s = \sqrt{\{1/[(1-S)^2+1]\}}$ is the slip factor of the rotor winding impedance change during stable rotation.
- $K_f = (2-S)$ is the frequency factor of the rotor winding impedance change.

This impedance frequency factor is only generated at the reactance part of the winding impedance and has no effect on the resistance part [1]. The derivation result of this equation is shown in Fig. 2, which is the complete equivalent circuit of the induction motor[1], depicted as Fig. 3b.

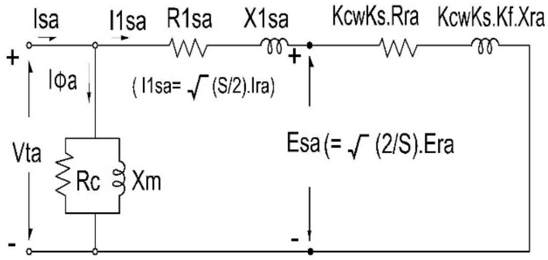


Figure 3b. Three-phase induction motor a-phase stator equivalent circuit diagram (When 0<S<1)

### *B. Energy Consumption Analysis*

All inputs from the external power supply of the induction motor provide energy ($3 \cdot V_{ta} \cdot I_{sa}$) . Subtracting the excitation energy required for the shunt of magnetizing part (can be measured through no-load test, $P_{Z\Phi a} + Q_{Z\Phi a}$), The remainder is exactly equal to the current $I_{1sa}$ flowing through the stator winding impedance ($R_{1sa} + X_{1sa}i$) generating energy) ($P_{1sa} + Q_{1sa}i$).

Additionally, this current flows through the equivalent impedance that the rotor impedance reference to the stator side, given by:

$[Z_{sa} = K_{cw} \cdot K_s \cdot (R_{ra} + K_f \cdot X_{ra}i)]$

The energy dissipation of this equivalent impedance (PEsa+QEsa) is actually the counter EMF of the stator phase winding transmitted to the total energy generated by the induced voltage and current of the rotor phase winding (Pra+Qra). In short, all the energy input by the external power supply is completely distributed to the stator and rotor, conforming to the principle of conservation of energy.

$$I1_{sa} = V_{ta}/\sqrt{[(R_{1sa} + K_{cw} \cdot K_s \cdot R_{ra})^2 + \left(X1_{sa} + K_{cw} \cdot K_s \cdot K_f \cdot X_{ra}\right)^2]}$$

$$I_{\Phi a} = V_{ta}/Z_{\Phi a}\ , \ I_{sa} = I_{\Phi a} + I1_{sa}$$

$P_s = 3 \cdot I1_{sa}^2 \cdot R_{1sa}$ --------- All active power consumed in the stator phase winding

$Q_s = 3 \cdot I1_{sa}^2 \cdot X_{1sa}$---------All reactive power consumed in the stator phase winding

$P_{ES} = 3 \cdot I1_{sa}^2 \cdot (K_{cw} \cdot Ks \cdot R_{ra})$=$3 \cdot I1_{sa}^2 \cdot \{(2/S) \cdot \sqrt{\{1/[(1-S)^2+1]\}} \cdot R_{ra}\}$
----All active power consumed in the equivalent impedance of stator phase counter EMF.

$Q_{ES} = 3 \cdot I1_{sa}^2 \cdot \left(K_{cw} \cdot K_s \cdot K_f \cdot X_{ra}\right)$=$3 \cdot I1_{sa}^2\{(2/S) \cdot \sqrt{\{1/[(1-S)^2+1]\}} \cdot (2-S) \cdot X_{ra}\}$
----- All reactive power consumed in the equivalent impedance of stator phase counter EMF.

$P_r = 3 \cdot I_{ra}^2 \cdot K_s \cdot R_{ra}$
$= 3 \cdot \left[\sqrt{(2/S)} \cdot I_{1sa}\right]^2 \cdot$

**Table 3.The corresponding magnetic pole phase torque and the resulting total torque change period (S=1/6)**

| θe | θm | -Sin(θm) | Fsa-r | Fra-r | Ta | Fsb-r | Frb-r | Tb | Fsc-r | Frc-r | Tc | T-total | REMARK |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | 0° | 0 | + | + | 0 | - | - | 0 | - | - | 0 | 0 | @ θm = 0° ~ 90° |
| 54° | 45° | - | + | + | - | + | + | - | - | - | - | -0.16875(Min.) | Repulsion segment |
| 108° | 90° | - | - | 0 | 0 | + | 0 | 0 | - | 0 | 0 | 0 | @θm = 90° ~180° |
| 162° | 135° | - | - | + | + | + | - | + | - | + | + | 0.16875(Max.) | Attraction segment |
| 216° | 180° | 0 | - | + | 0 | + | - | 0 | + | - | 0 | 0 | @ θm = 180° ~270° |
| 270° | 225° | + | - | + | - | - | + | - | + | - | - | -0.16875(Min.) | Repulsion segment |
| 324° | 270° | + | + | 0 | 0 | - | 0 | 0 | + | 0 | 0 | 0 | @ θm = 270° ~360° |
| 378° | 315° | + | + | + | + | - | - | + | - | - | + | 0.16875(Max.) | Attraction segment |
| 432° | 360° | 0 | + | + | 0 | + | + | 0 | - | - | 0 | 0 | @ θm = 360° ~450° |
| 486° | 405° | - | - | - | - | + | + | - | - | - | - | -0.16875(Min.) | Repulsion segment |
| 540° | 450° | - | - | 0 | 0 | + | 0 | 0 | + | 0 | 0 | 0 | @ θm = 450° ~640° |
| 594° | 495° | - | - | + | + | - | + | + | + | - | + | 0.16875(Max.) | Attraction segment |
| 648° | 540° | 0 | + | - | 0 | - | + | 0 | + | - | 0 | 0 | @ θm = 540° ~630° |
| 702° | 585° | + | + | - | - | - | + | - | + | - | - | -0.16875(Min.) | Repulsion segment |
| 756° | 630° | + | + | 0 | 0 | - | 0 | 0 | - | 0 | 0 | 0 | @ θm = 630° ~720° |
| 810° | 675° | + | + | + | + | + | + | + | - | - | + | 0.16875(Max.) | Attraction segment |
| 864° | 720° | 0 | - | - | 0 | + | + | 0 | - | - | 0 | 0 | @ θm = 720° ~810° |
| 981° | 765° | - | - | - | - | + | + | - | + | + | - | -0.16875(Min.) | Repulsion segment |
| ↓ | ↓ | | | | | | | | | | | ↓ | ↓ |

**Remarks:1. θe=ωe.t, θm=(1-S)ωe.t 2. Fxx-r = Fxx-resultant (e.g. Fsa-r= Fsa-resultant ). 3. T-total=Ta+Tb+Tc**
**4.Ta=-( Fsa-r).( Fra-r).Sin(1-S).ωet, Tb=-( Fsb-r).( Frb-r).Sin(1-S).ωet, Tc=-( Fsc-r).( Frc-r).Sin(1-S).ωet**

$$\sqrt{}\{1/[(1-S)^2+1]\}\cdot \mathrm{R_{ra}}$$
$$= 3\cdot(2/S)\cdot I1_{\mathrm{sa}}^2\cdot\{\sqrt{}\{1/[(1-S)^2+1]\}\cdot \mathrm{R_{ra}}\}$$

-----All Active Power transmitted from the stator to the rotor

$$Q_r = 3\cdot I_{ra}^2\cdot K_s\cdot K_f\cdot X_{ra}$$
$$= 3\cdot\left[\sqrt{}(2/S)\cdot I_{1sa}\right]^2\cdot$$
$$\sqrt{}\{1/[(1-S)^2+1]\}\cdot(2-\mathrm{S})\cdot \mathrm{X_{ra}}$$
$$=3\cdot \mathrm{I1_{sa}^2}\left\{\left(\frac{2}{S}\right)\cdot\left\{\frac{1}{[(1-S)^2+1]}\right\}\cdot(2-\mathrm{S})\cdot \mathrm{X_{ra}}\right\}$$

-----All rective Power transmitted from the stator to the rotor

The above calculation considers the total amount under balanced three-phase conditions and obtains $\mathrm{P_{Es}} = P_r$ and $\mathrm{Q_{Es}} = Q_r$. This means that all the active power and reactive power consumed by the equivalent impedance of the stator (counter EMF) will be transmitted to the rotor via the air gap flux.

## V. ANALYSIS OF INDUCTION MOTOR TORQUE GENERATION

According to the rotating electrical machine torque theory [2], whether from the coupled-circuit viewpoint, it is known that the rotating torque, $\mathrm{T} = -\left(\frac{\mathrm{P}}{2}\right)\cdot \mathrm{L_{sr}}\cdot I_s\cdot I_r\cdot Sin[\left(\frac{P}{2}\right)\cdot\theta_m]$, or from the magnetic field viewpoint, for the rotation torque of a P-pole machine, the torque is given by:
T= -(P/2 ).(μo/2) . (Dι/g).Fs.Fr.Sinδsr
, This can also be expressed: $\mathrm{T} = -(P/2)\cdot(\pi/2)\cdot(\mu_o/g)\cdot(D_1/g)\cdot F_s\cdot F_{sr}\cdot Sin(\delta_s)$ , or $\mathrm{T} = -(P/2)\cdot(\pi/2)\cdot(\mu_o/g)\cdot(D_1/g)\cdot F_r\cdot F_{sr}\cdot Sin(\delta_r)$

In short, the torque is generated by the interaction between the two corresponding MMFs generated by the stator winding and the rotor winding. The negative sign in the formulas indicates that the torque is generated due to the two corresponding magnetic axis of the stator winding and the rotor phase winding tend to be align. The generation of individual MMFs is caused by the current flowing through the reactance components of the winding impedance, regardless of their resistance components. This part of the energy consumption ($\mathrm{Q_s}$ and$\mathrm{Q_r}$) is required to generate the MMF and interact to produce torque.

The current flowing through the resistive component of the individual winding impedance only produce waste heat energy (Ps and Pr), which cannot do work. Therefore, after deducting the energy required by the excitation shunt, all the energy consumed by the stator and rotor of the induction motor whether it is active power (copper loss of electric field energy) or reactive power (magnetic field electromagnetic energy) is used to generate energy. It is just the energy generation of timing, space, electric field or magnetic field are different.

In addition, based on the magnetic field viewpoint of the Torque theory of rotating electrical machines,

$$\mathrm{T} = -\left(\frac{\mathrm{P}}{2}\right)\cdot\left(\frac{\mu_o}{\mathrm{g}}\right)\cdot\left(\frac{D_1}{\mathrm{g}}\right)F_s\cdot F_{sr}\cdot Sin(\delta_{sr})$$
$$= K_r\cdot F_s\cdot F_r\cdot Sin(\delta_{sr})$$

In the above formula; $K_r = (P/2)\cdot(\frac{\mu_o}{2})\cdot\left(\frac{D_l}{g}\right)$ is a fixed value. Bring "(7)"~"(12)" to the corresponding Fs and Fr respectively, and the corresponding magnetic axis angle difference between the phase winding of stator and rotor that change over time are brought in for calculation to understand the changes in the magnitude of the torque (design an Excel calculation worksheet of the time-series changes for each phase torque and the combined total torque of an induction motor ).

Ultimately, The overall torque change of the induction motor after summing up the individual torques of the three phases; it is found that the stator magnetic axis of the motor is fixed, corresponding to each complete rotation(the mechanical angle is 2π/360°, the first to fourth quadrants) of the rotor magnetic axis, the induction motor torque will be generated from zero → Minimal (negative torque) →zero →Maximal (positive torque) →zero →Minimal (negative torque) → zero → Maximal (positive torque) → zero. Each subsequent rotation repeats the torque change of the previous two-cycle wave again (i.e. Table 3. and Figure 4.).

The torque change of these two periodic waves; that is, from the initial negative torque in the first quadrant (repulsive force) → positive torque in the second quadrant (attract force) → negative torque in the third quadrant (repulsive force) → positive torque in the fourth quadrant (attract force).

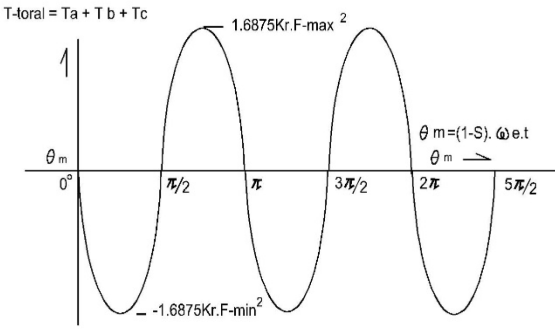


Figure 4. The simulation waveform for torque change of the rotor.

## VI. THE CASE ANALYSIS OF CALCULATION THE EQUIVALENT CIRCUIT CHARACTERISTICS, IMPEDANCE , ENERGY CONSUMPTION

### A. *The Case Analysis Of Induction Motor Factory Test*

Based on the results of four kinds of induction motor factory test (Table 4.) [4] and the applicable data are calculated in accordance with Kirchhoff's voltage and current law and Ohm's law. Including line current, voltage, impedance, the actual values of each component, the required energy consumption and torque generated of the equivalent circuit under various load conditions.

Table 4. Test Report of Induction Motor

| Rating: | | | | | |
|---|---|---|---|---|---|
| Output:650KW | Phase: 3 | | Frequence:60Hz | | |
| Voltage:3300V | Poles: 4 | | Time Rating:S1 | | |
| Current:132.4A | Speed: 1785 rpm | | S.F. : 1.15 | | |
| Type of Rotor: CAGE | Frame No: N450LSZ | | Ins. Cless:F | | |
| Serial No.:89100245 | Manufacturer: TATUNG , TAIWAN | | | | |
| Characteristics test : (Test date: Octber 19, 2018 ) | | | | | |
| | | Hz | Volts | Amps | Watts |
| 1.Locked Rotor Test | | 60 | 717.3 | 131.27 | 18610 |
| 2.No Load Test(Rat. Volts) | | 60 | 3300 | 31.12 | 15860 |
| 3.No Load Test(@L.R. Volts) | | 60 | 717.3 | 6.7643* | 749* |
| 4.Load Characteristics test : | | | | | |
| Load(%) | KW | Current(A) | Slip(%) | P.F.(%) | Eff.(%) |
| 25% | 162.5 | 45.6 | 0.196 | 68.7 | 90.8 |
| 50% | 325 | 71.1 | 0.396 | 84.4 | 94.8 |
| 75% | 487.5 | 100.3 | 0.606 | 88.6 | 95.9 |
| 100% | 650.5 | 131.8 | 0.829 | 89.6 | 96.3 |
| 125% | 812.5 | 165.3 | 1.074 | 89.2 | 96.4 |

Remark: 3. *No Load Test (@L.R. Volts. ---By calculated

1) *Locked-Rotor Test @Rat. Current 、Slip S=1 And* $I_{sa} = I_{\phi a} + I_{1sa}$

Reference Fig. 3a and reported data from Table 4, $V_{t(p-p)} = 717.3$ Volts, $I_{sa-1} = 131.27$ Amps & $P_{total-3\Phi} = 18610$ Watts

Thus calculated, $\sqrt{3 \cdot V_t \cdot I_{sa} \cdot Cos\theta} = P_{total-3\Phi}$、$\sqrt{3 \cdot V_t \cdot I_{sa} \cdot Sin\theta} = Q_{total-3\Phi}$ , $Sin\theta = \sqrt{(1 - Cos\theta^2)}$

$3 \cdot (I_{sa-1})^2 \cdot R_{eq-1} = P_{total-3\Phi}$,

$3 \cdot (I_{sa-1})^2 \cdot X_{eq-1} = Q_{total-3\Phi}$ &

$$I_{sa} = (V_t/\sqrt{3})/Z_{eq-1}$$

$$= (V_t/\sqrt{3})/(R_{eq-1} + X_{eq-1} \cdot i)$$

Obtain, $Cos\theta = 0.114$

$Sin\theta = 0.9935$

$Q_{total-3\Phi} = 162024$ Vars

$$I_{sa-1} = 14.9791 - 130.4126i$$

$$= 131.27\angle -83.45°$$

Req-1 = 0.3600 ---the real part of $\{Z_{\Phi a} || [(R_{1sa} + R_{ra}) + (X_{1sa} + X_{ra}) \cdot i]\}\}$

Xeq-1 = 3.3142---- the imaginary part of $\{Z_{\Phi a} || [(R_{1sa} + R_{ra}) + (X_{1sa} + X_{ra})i]\}\}$

$Z_{eq} - 1 = 0.3600 + 3.1342i$

(" || " means parallel connection)

2) *No-Load Test @Rat. Volts, Slip* $S = 0$ *and* $I_{sa} = I_{\Phi a} + I_{1sa}$ , *but* $I_{1sa} = 0$

Same reference Fig. 3a and reported data Table 4, $V_{t(p-p)}$=3300Volts, $I_{sa-2} = 31.12$ Amp & $P_{total-3\Phi} = 15860$ Watts，

As calculated above for the locked-rotor test, obtain , $Cos\theta = 0.0892$

$Sin\theta = 0.9960$

$Q_{total-3\Phi} = 177166$Vars

$I_{sa-2} = I_{\Phi a} = 2.7748 - 30.9960i$

$= 31.12\angle -84.88°$

Due to No-Load Test, $S = 0$ , causes

$[R_{1sa} + K_{cw} \cdot K_s \cdot R_{ra}] = \infty$ &

$[X1_{sa} + K_{cw} \cdot K_s \cdot K_f \cdot X_{ra}]i = \infty$

Therefore, it is calculated;

$R_{eq-2} = 5.4589$--------- the real part of $Z_{\Phi a}$

$X_{eq-2} = 60.979$-------- the imaginary part of $Z_{\Phi a}$

$Z_{eq-2} = 5.4589 + 60.9790i = Z_{\Phi a}$

3) *No-Load Test ,step-down @* $V_{t(p-p)=717.3}$ *Volts, Slip(* $S) = 0$ *and* $I_{sa} = I_{\Phi a}$ ，$I1_{sa} = 0$

Reference Fig.3a and reported data from Table 4, $V_{t(p-p)} = 717.3$ Volts、$I_{sa-3} = 6.744$Amps & $P_{total-3\Phi} = 749.34$ Watts,

As the above calculation, obtain,

$Cos\theta = 0.0892$

$Sin\theta = 0.9960$

$Q_{total-3\Phi} = 8370.56$Vars

$I_{sa-3} = I_{\Phi} = 0.6031 - 6.7374i$

$= 6.7644\angle -84.88°$

Same as *2) No-load test*, Since $S = 0$, cause $[R_{1sa} + K_{cw} \cdot K_s \cdot R_{ra}] = \infty$ and

$[X_{1sa} + K_{cw} \cdot K_s \cdot K_f \cdot X_{ra}]i = \infty$

Therefore, it is calculated,

$R_{eq-3} = 5.4589$ -------- the real part of $Z_{\Phi a}$

$X_{eq-3} = 60.9790$ ------the imaginary part of $Z_{\Phi a}$

$Z_{eq-3} = 5.4589 + 60.9790i = Z_{\Phi a}$

*4) Load characteristic test (Load Test @ Rat. Volts, Slip $0 < S < 1$ and $I_{sa} = I_{\Phi a} + I_{1sa}$)*

Here, we only report the data based on the 100% load (650KW) test in (Table 4) and refer to the Calculation in Fig. 3b. The calculation results for other load conditions (125%, 75%, 50% and 25%) are also listed in Appendix A.

Reference Fig.3b and reported data from Table- 4, $V_{t(p-p)} = 3300$Volts, $I_{sa} = 131.8$Amps, $S = 0.00829$ and P.F. $(\text{Cos}\theta) = 89.6\%$,

As the above calculation; obtain,

$\text{Sin}\theta = 0.44405$

$P_{total-3\Phi} = 674991.01$Watts

$Q_{total-3\Phi} = 334522.87$Vars

$I_{sa-4} = 118.0928 - 58.5263i = 131.80\angle - 26.3628°$

$R_{eq-4} = 12.9523$-------------the real part of $\left\{\begin{matrix}[R_{1sa} + K_{cw}\cdot K_s\cdot R_{ra}]\\+[X_{1sa} + K_{cw}\cdot K_s\cdot K_f\cdot X_{ra}]i\end{matrix}\right\} || Z_{\Phi a}$

$X_{eq-4} = 6.4191$-----------the imaginary part of $\left\{\begin{matrix}[R_{1sa} + K_{cw}\cdot K_s\cdot R_{ra}]\\+[X_{1sa} + K_{cw}\cdot K_s\cdot K_f\cdot X_{ra}]i\end{matrix}\right\} || Z_{\Phi a}$

$Z_{eq-4} = 12.9523 + 6.4191i$---- The total impedance of phase-a of the motor

*B. The Calculation Method Of Individual Impedance Values Of Stator And Rotor*

The calculation results of the four test data in Chapter VI.*A*.1) ~ 4) :

*1)* Chapter VI.*A*.1), the rated current blocking test results Zeq-1 and Isa-1 subtract the step-down no-load test results Zeq-3 and Isa-3 in Chapter VI. *A*.3). These calculations obtain the values of impedance Zeq-LR and current I1s-LR of the rated current blocking test, which only includes the load component (excluding the excitation shunt). The current vector Heyland circle diagram[5] is shown in Fig. 5.

$$(Z_{eq-1}) - (Z_{eq-3}) = \frac{1}{\left[\left(\frac{1}{Z_{eq-1}}\right) - \left(\frac{1}{Z_{eq-3}}\right)\right]}$$

$$= \frac{1}{\left\{\left[\frac{1}{0.3600 + 3.1342\,i}\right] - \left[\frac{1}{5.4589 + 60.9790i}\right]\right\}}$$

$$= 0.3840 + 3.3039i$$

$$= (R_{1sa} + R_{ra}) + (X_{1sa} + X_{ra})i$$

$$= Z_{eq-LR} \qquad (29)$$

$$(I_{sa-1}) - (I_{sa-3}) = (14.9791 + 130.4126i) - (0.6031 + 6.7374i) = 14.376 + 123.6752i = 124.51\angle - 83.37° = I1_{sa-LR}$$

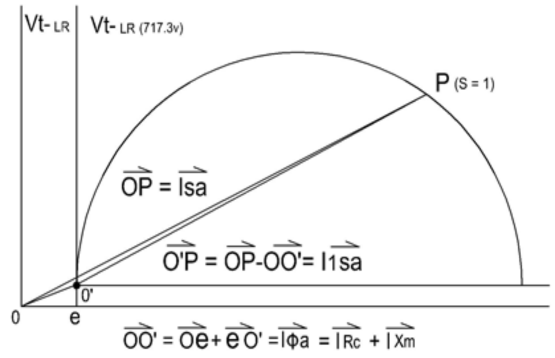


Figure 5. The current vector of Heyland circle diagram (Locked-Rotor Test @Rat. Current)

*2)* Reference Chapter VI.A.4), the rated voltage load characteristic test results Zeq-4 and Isa-4 subtract the rated voltage no-load test results Zeq-2 and Isa-2 in Chapter VI.A.2). These calculations obtain the values of impedance Zeq-100%Load and current I1sa-100%Load of the rated voltage 100%Load test, which only includes the load component (excluding the excitation shunt). The current vector Heyland circle diagram is shown in Fig. 6.

$$(Z_{eq-4}) - (Z_{eq-2}) = \frac{1}{\left[\left(\frac{1}{Z_{eq-4}}\right)-\left(\frac{1}{Z_{eq-2}}\right)\right]} = \frac{1}{\left\{\left[\frac{1}{12.9523+6.4191\,i}\right]\right\}} - \left[\frac{1}{5.4589+60.9790\,i}\right] = 15.6309 + 3.7316i = [(R1_{sa} + K_{cw}\cdot K_s\cdot R_{ra}) + (X1_{sa} + K_{cw}\cdot K_s\cdot K_f\cdot X_{ra})i] = Z_{eq-100\%Load} \qquad (30)$$

$(I_{sa-4}) - (I_{sa-2}) = (118.0928\text{-}58.5263i)\text{-}(2.7748\text{-}30.9960i) = 115.3180 - 27.5303i = 118.5586\angle - 13.43° = I_{1sa-100\%Load}$

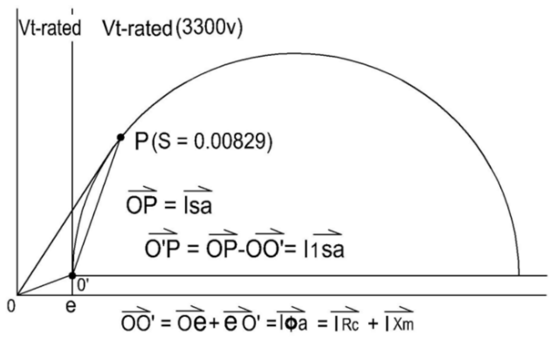


Figure 6. The current vector of Heyland circle diagram (100%Load test @Rat. Voltage )

Because the slip(S) = 0.00829 at 100%Load, so

$K_{cw} = \frac{2}{S} = 241.2545235$--- The period factor of the rotor winding impedance referenced to the stator side.

$K_s = \sqrt{\left\{\frac{1}{(1-S)^2} + 1\right\}} = 0.7100$----The slip factor of the Rotor winding impedance change during rotation.

$K_f = (2 - S) = 1.99171$---The frequency factor of Reactance change in rotor phase winding

Then, [“(30)”–“(29)”] =

$[(R_{1sa} + K_{cw} \cdot K_s \cdot R_{ra}) + (X_{1sa} + \mathrm{K_{cw}} \cdot \mathrm{K_s} \cdot \mathrm{K_f} \cdot \mathrm{X_{ra}})\mathrm{i}] - [(R_{1sa} + R_{ra}) + (X_{1sa} + \mathrm{X_{ra}})\mathrm{i}] = (K_{cw} \cdot K_s \cdot R_{ra} - R_{ra}) + (K_{cw} \cdot K_s \cdot K_f \cdot X_{ra} \cdot X_{ra})i = (K_{cw} \cdot K_s - 1) \cdot R_{ra} + (K_{cw} \cdot K_s \cdot K_f - 1) \cdot X_{ra}i$ =(15.6309+3.7316i) - (0.3840 + 3.3039i) = 15.2469 +0.4277i

The solution is Rra =0.08953
and $\mathrm{X_{ra}}i = 0.00126i$ ,
So,

$$\mathrm{Z_{ra-100\%Load}} = K_s \cdot R_{ra} + K_s \cdot K_f \cdot X_{ra}i = 0.06357 + 0.00178i$$

and

$$\mathrm{Z_{Esa-100\%Load}} = K_{cw} \cdot Z_{ra} = 15.336 + 0.4289i$$

Then get it from ["(29)" $- (\mathrm{R_{ra}} + \mathrm{X_{ra}}i)$] ,

$$R_{1sa} + X_{1sa}i = 0.2945 + 3.3026i$$

Therefore, according to Table 4. The test report data and the calculation results in Chapter VI.*A~B*, detailed $\mathrm{R1_{sa}}$ , $\mathrm{X1_{sa}}$ , $(\mathrm{K_{cw}} \cdot K_s \cdot R_{ra})$ and $(\mathrm{K_{cw}} \cdot K_s \cdot K_f \cdot X_{ra})$ values can be obtained, the measurement value of Power factor (P.F.) and slip (S) needs to be quite accurate. Special attention should be paid to the number of decimal points in the calculation process, otherwise the calculation results will be affected. Calculations for other loads, such as 25%, 50%, 75% and 125%, can be handled as well.

### *C. Actual Verification Of Equivalent Circuit And Induction Motor Characteristics*

To understand the effect of the induction motor on its supply circuit and its own characteristics under various load conditions, this paper reports the actual factory test result data of an induction motor. The data is presented in Table 4. The actual calculation results are shown in this chapter and in Appendix A (Table 5(a), Table 5(b), and Table 5(c)). These results cover various load conditions, equivalent circuit component parameters, energy distribution, and output torque data.

Additionally, it is verified that under each % Load condition, the counter EMF plus the stator winding voltage drop equals the external input voltage. This verification is presented in Table 5(c).

## VII. DISCUSSION AND CONCLUSION

This study reveals that the voltages and currents on the stator and rotor sides exhibit different frequency cycles, with the rotor side showing variations in equivalent impedance parameters at different slip speeds. The most significant finding is that the new equivalent circuit model can better comply with energy conservation principles and Thevenin's theorem. Additionally, the calculation analysis of the new equivalent circuit model reveals that the energy to generate torque comes from the reactive power consumed on both the stator side and the rotor side, which generate separate magnetic fields that interact with each other. In summary, all the energy input from the external power supply, whether active power (copper loss) or reactive power (electromagnetic energy), contributes to generating the magnetic field, with differences only in time, space, electric field or magnetic field. In terms of Electrodynamics, active power does not contribute to work, whereas reactive power is useful energy for generating work.

These results strongly support for Faraday's law of electromagnetic induction and enhance our understanding of the magnetomotive force variations in induction motors under various load conditions. Specifically, the findings validate the mathematical formulas expressing the magnetomotive force (MMF) changes in stator and rotor windings.

The implications of these findings are substantial for the design and efficiency improvement of induction motors. By accurately calculating the rotor's equivalent impedance referred to the stator side, energy losses can be reduced, and torque output efficiency can be improved, leading to more efficient motor designs.

Compared to the work of Fitzgerald and Kingsley (2014) on “Electric Machinery,” this study provides a more detailed quantification of the impact of slip on rotor voltage and current, offering new insights for advanced motor design.

Since the stator and rotor windings of induction motors are subject to changes in AC electric and magnetic fields, while the rotor windings of synchronous motors (or even synchronous generators) have an additional DC magnetic field, it is more complicated to combine these effects. Why does the rotor winding of a synchronous machine change under any excitation condition? It is feedback reactive power (-Qr), unlike the stator winding which continuously consumes reactive power (+Qs). After reviewing relevant discussions, no equivalent circuit model that complying with Thevenin's theorem has been studied. Therefore, further research and exploration on this device are expected to verify or expand new findings.

In summary, this study's detailed mathematical modeling and experimental data highlight the specific rules governing voltage and current variations in induction motor stators and rotors. These insights are crucial for enhancing motor design and application, demonstrating the practical value of the proposed new equivalent circuit model.

## REFERENCE

[1] Fitzgerald & Kingsley on Electric Machinery --2014/ 7th-ed/ Chapter 6〔6.3〕, p-351~p361

[2] Fitzgerald & Kingsley on Electric Machinery – 2014/ 7th-ed/ Chapter 4〔4.6~7〕, p-218~p241

[3] B. Hague, “The mathematical treatment of the magnetomotive force of armature windings”, Volume 55, Issue 268, July 1917, p. 489 – 514 /DOI: 10.1049/jiee-1.1917.0032 , Print ISSN 0099-2887, Online ISSN 2054-0612

[4] Tatung Motor manufacture company Factory test report (p1~p11 ):(Ser.No.89100245 /3Φ, 4P, 3.3kv, 650kw), test date: Oct.19, 2018

[5] Heyland, A. A Graphical Method for the Prediction of power Transformers and Polyphase Motors) (PDF). ETZ:561-564. 1894

[6] Fitzgerald & Kingsley on Electric Machinery— 2014/ 7th-ed/ Chapter 2〔2.4〕, p-74~p78

[7] J. E. Brittain, "Thevenin's theorem," in *IEEE Spectrum*, vol. 27, no. 3, pp. 42- , March 1990, DOI: 10.1109/6.48845.

# APPENDIX A

Table 5(a),(b)&(c) Under various load conditions, equivalent circuit component parameters, energy distribution and output torque data

| Load% | I1sa/(rms) (A) | (P+Q)-3Φ | (Ps+Qs)-3Φ | (Pr+Qr)-3Φ | Cosθ | Sinθ | θ |
|---|---|---|---|---|---|---|---|
| L.R. Test | 14.38+123.7i/(124.5) | 17861+153654i | 13725+153593i | 4135+61i | 0.1155 | -0.9933 | -83.37° |
| 125%Load | 144.7+43.73i/(151.1) | 826916+249924i | 20223+226316i | 806692+23608i | 0.9572 | -0.2893 | -16.82° |
| 100%Load | 115.3+27.53i/(118.6) | 659131+157357i | 12419+139268i | 646712+18089i | 0.9727 | -0.2322 | -13.43° |
| 75%Load | 86.09+15.51i/(87.48) | 492076+88661i | 6743+75820i | 485333+12841i | 0.9842 | -0.1773 | -10.21° |
| 50%Load | 57.23+7.14i/(57.68) | 327134+40799i | 2927+32962i | 324207+7837i | 0.9923 | -0.1283 | -7.11° |
| 25%Load | 28.55+2.14i/(28.63) | 163199+12229i | 720+8123i | 162479+4106i | 0.9972 | -0.0747 | -4.29° |
| No-Load-1 | 2.77+31.00i/(31.12) | 15860+177166i | 0 | 0 | 0.0892 | -0.9960 | -84.88° |
| No-Load-2 | 0.60+6.74i/(6.76) | 749+8371i | 0 | 0 | 0.0892 | -0.9960 | -84.88° |

**Remarks: 1. No-Load-1 will be No-Load test @Rated -voltage. 2. No-Load-2 will be No-Load test @L.R. Test -voltage. 3. (P+Q)-3Φ =[(Ps+Qs)-3Φ]+[ (Pr+Qr)-3Φ] 4. θ is the angle of stator phase winding current (i.e. Ī1sa∠θ)**

**(a)**

| Load% | √3.Vt.Isa | Z1sa=R1sa+X1sai | Zra=ks.Rra+ks.kf.Xrai | ZEsa=(2/S).Zra | S | ωm | T-total |
|---|---|---|---|---|---|---|---|
| L.R. Test | 18610+162025i | 0.2951+3.3026i | 0.08892+0.00131i | 0.0889+0.0013i | 1 | 0 | 0 |
| 125%Load | 842776+427090i | 0.2951+3.3026i | 0.06322+0.00185i | 11.772+0.3445i | 0.01074 | 186.5 | 1340 |
| 100%Load | 674691+335130i | 0.2945+3.3026i | 0.06357+0.00178i | 15.336+0.4289i | 0.00829 | 186.9 | 842 |
| 75%Load | 507936+265827i | 0.2937+3.3027i | 0.06405+0.00169i | 21.241+0.5593i | 0.00606 | 187.4 | 473 |
| 50%Load | 342994+217965i | 0.2933+3.3028i | 0.06432+0.00155i | 32.486+0.7853i | 0.00396 | 187.7 | 217 |
| 25%Load | 179059+189359i | 0.2926+3.3028i | 0.06474+0.00164i | 66.063+1.6695i | 0.00196 | 188.1 | 65 |
| No-Load-1 | 15860+177166i | gc =5.5489 | bm =60.9790i | - | 0 | - | - |
| No-Load-2 | 749+8371i | gc =5.5489 | bm =60.9790i | - | 0 | - | - |

**Remarks: 1.Total input-power =√3.Vt.Isa 2. ωm = ωe .(1-S) 3. T-total (Newton-metre) =(Qs + Qr) /ωm**

**(b)**

| Load% | Vta | V1sa= I1sa.(R1sa+X1sai) | Esa = I1sa.ZEsa /(rms) | Era /(rms)=Ira. Zra | Esa / Era |
|---|---|---|---|---|---|
| L.R. Test | (717.3+0i)/√3 | 412.69+10.978i/(412.8) | 1.4401-10.9786i/(11.07) | 1.44-10.9786i/(11.07) | 1 |
| 125%Load | (3300+0i) /√3 | 187.10+466.89i/(501.1) | 1718.15-464.89i/(1779.9) | 125.91-34.07i/(130.43) | 13.65 |
| 100%Load | (3300+0i) /√3 | 124.89+372.75i/(393.1) | 1780.37-372.35i/(1818.9) | 114.62-23.99i/(117.11) | 15.53 |
| 75%Load | (3300+0i) /√3 | 76.52+279.78/(290.05) | 1828.74-279.78i/(1850.0) | 100.66-15.40i/(101.83) | 18.17 |
| 50%Load | (3300+0i) /√3 | 40.36+186.94/(191.25) | 1864.89-186.94i/(1874.2) | 82.98-8.32i/(83.40) | 22.47 |
| 25%Load | (3300+0i) /√3 | 15.42+93.68i/(94.94) | 1889.83-93.68/(1892.16) | 59.16-2.93i/(59.23) | 31.94 |

**Remarks: 1. Vta=V1sa+ Esa =(3300+0i)/√3 2. Esa= I1sa .(kcw.ks.(Rra+kf.Xrai) 3. When 0<S <1，Esa /Era =√(2/S)**

**(c)**